

# Industrial validation of the functional performance equation for ball milling and pebble milling circuits

## Introduction: the initial breakthrough in understanding

The author's interest in grinding was sparked at Allis-Chalmers, where Fred C. Bond and Chester A. Rowland led the company's grinding process technology. These icons of grinding-mill process engineering provided a rigorous mill sizing for application discipline, which today remains the world standard (Bond, 1961; Rowland, 2002).

Having established a standard relationship for comparison of lab (predicted) versus plant (actual) energy use, a plant grinding circuit efficiency metric was also created (Rowland, 1976). The ratio of plant operating to ore work index provided a quantitative measure of overall grinding-circuit efficiency. Using this, a metallurgist could explore whether changing a circuit design or operating variable increased (or decreased) efficiency in the plant. Bond work index analysis is based on energy usage. Because energy costs and the closely related costs for grinding media dominate grinding-circuit operating costs, changes (process improvements) requiring capital expenditure can then be financially justified.

The author also learned slurry pump and cyclone process application engineering methods through employment with these equipment manufacturers. It was learned how the cyclone water and solids mass split and how separation performance curves are calculated from the feed and product percent solids and size distributions. It was also learned that the cyclone selection procedure uses the same relationships. Given the cyclone feed, the desired products are achieved by choosing the cyclone (dimensions and operating conditions, including those provided by the pump) that provides the right particle separation curve and water split.

But what was also extraordinary was that, in the case of a closed-circuit operation, the pump and cyclones could be chosen to not only manipulate the cyclone product size distributions but also the cyclone feed size distribution

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(McIvor, 1984). The same engineering procedures that were used to select cyclones and pumps for a new application could be used to make cyclone and pump changes in an existing plant to manipulate the internal size distributions and related mass flows (the circulating load) of a closed grinding circuit.

Combining this new-found knowledge with the critical and well-known relationship between grinding circuit productivity and the circulating load ratio (for example, see the references by Davis, 1925, and Gaudin, 1939) led to the realization that there existed a valuable opportunity to improve grinding circuit efficiency. An observed poor (low) circulating-load ratio could be increased by suitable pump and cyclone changes — thus increasing circuit efficiency. This, in turn, could be verified by Bond work index analysis and the related operating cost savings used to justify the cost of the plant improvements.

But, although the circulating load effect published by Davis and Gaudin was broadly known, it was not at all understood. The question was: "Why is circuit performance so drastically affected by circulating load ratio?" The literature, including Davis's and Gaudin's, offered no explanation.

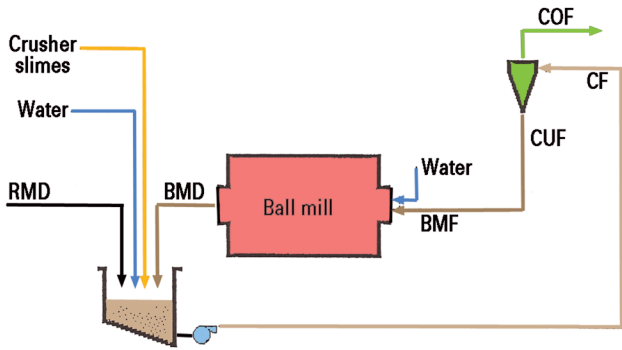
It was while examining size-distribution data from two plants that had about the same circuit product sizes (P80s) but different circulating load ratios — one extremely low (<150 percent) and one extremely high (>500 percent) — that a revelation occurred. Seeing very different size distributions into and out of the two ball mills, the reason for the huge difference in circuit efficiencies shown by the Davis relationship suddenly became clear. With the low circulating load, the ball mill was relatively full of fines (P80 product size or finer material). Therefore, most of the grinding energy of the mill was being wasted. With the high circulating load, the mill was relatively full of coarse material and relatively little

## Abstract

*The "functional performance equation for ball milling" was first presented in 1988. It has now been used successfully in a substantial number of mineral-processing plants. This powerful, yet simple, tool provides a new level of understanding of closed circuit grinding. It shows how grinding circuit efficiency is really comprised of two distinct efficiencies. It demonstrates how circuit production rate is a factor of these two efficiencies, energy input and the grindability of the ore. And it links circuit performance to design and operating variables that can be manipulated. Thus, it provides an effective strategy for making plant improvements. This paper covers the derivation and industrial validation of this equation. As part of a grinding process engineering system that also incorporates suitable metrics and process control, all operators can use this tool to improve and manage the performance of their grinding operations with clarity and confidence.*

**FIGURE 1**

**Simplified schematic of the Selbaie ball mill circuit.**



of its energy was being wasted. The effect that circulating load has on circuit efficiency was related to the corresponding proportions of coarse versus fine material in the mill.

Subsequent investigations showed that the arithmetic average of the percentage of “coarse” material (the circuit P80 being the typical cut off size) in the mill feed and discharge size distributions yields a meaningful, quantitative measure of the percentage of coarse material in the mill (McIvor, 1988). The validation of this and subsequent aspects of this new method for improving plant grinding performance is the subject of this paper.

### Derivation of the functional performance equation

**Circuit “classification system efficiency” and “effective mill power” defined.** With reference to the standard closed grinding circuit shown in Fig. 1, the above-described observations lead to the following definition of ball mill circuit “classification system efficiency” (CSEff). It is the percentage of “coarse” material in the ball mill, relative to the target grind size, typically the circuit target P80. The “coarse” material is targeted for further grinding, while the “fines” have reached target product size or finer. It follows that the circuit CSEff is also the relative percentage of mill power being expended on “coarse” material versus “fines.” Similar to the efficiency of a drive component, such as a motor, it is the percentage of the mill energy that is delivered and used for the intended purpose. It is noteworthy that a higher CSEff also means less overgrinding of “fines” and, therefore, improved recovery of valuable minerals in downstream processes such as flotation (McIvor and Finch, 1991).

The CSEff can be calculated as the arithmetic average of “coarse” material in the ball mill feed and discharge. It represents the net outcome of all the factors that create the size distributions of the material entering and leaving the mill. There are two key factors in this regard. The first is the classifier performance. It controls the percentage of “fines” versus “coarse” material reporting to the cyclone underflow/mill feed. The second factor is the length of time in the mill. This determines the amount of “fines” that accumulate during each pass through the mill. Low circulating load equates to long mill residence time and the buildup of fines. Another factor that plays a role is the breakage characteristic (the tendency to create fines during a breakage event) of the ore. But it is the combination of classifier performance and circulating load ratio that is the key to determining the net outcome of circuit

performance in terms of CSEff.

The “effective mill power” (EMP) can then be defined relative to the total mill power (TMP). EMP is the percentage of total mill power draw delivered to the “coarse” ore and is defined as

$$EMP = TMP \times CSEff \quad (1)$$

### The ball mill circuit functional performance equation.

A practical measure of a given circuit’s productivity (as used by Davis, for example) is the relative production rate of new product (PRNP) size material (in Davis’s case, -106  $\mu\text{m}$  or -150 mesh). The production of new product (or “fines”) comes about from the application of power to the “coarse” material. This is the “effective mill power” (EMP) as defined above. So it can be stated that the production rate of “fines” in the circuit equals the specific grinding rate of “coarse” material (SGRC), i.e., per unit energy applied to it, times the amount of power being applied to it (the “effective mill power”). This can be written as

$$PRNP = EMP \times SGRC \quad (2)$$

Substituting EMP from Eq. (1) gives

$$PRNP = TMP \times CSEff \times SGRC \quad (3)$$

The specific grinding rate of coarse material (SGRC) will depend on two factors, the grindability of the ore (the opposite of its resistance to size reduction) and the efficiency of usage of the energy that is applied to the coarse particles. This efficiency will be determined by factors, such as grinding ball sizing and percent solids in the mill. A standardized lab grindability test can be carried out on the coarse material that is being fed to the mill. Then, the ratio of the plant mill specific grinding rate (SGRC) of coarse material to the standardized lab mill-grinding rate (LabGr) of coarse material will be a relative measure of the efficiency of usage of this energy that is being applied to coarse material (the efficiency of the mill grinding environment).

So, to incorporate the material’s grindability into Eq. (3), divide and multiply the specific grinding rate of coarse particles by the measured lab grindability of the same material as follows

$$PRNP = TMP \times CSEff \times SGRC / \text{LabGr} \times \text{LabGr} \quad (4)$$

One can then define the ratio of plant to lab grinding rates as the relative “ball mill grinding efficiency” (BMGEff) as follows

$$SGRC / \text{LabGr} = \text{BMGEff} \quad (5)$$

Substituting in Eq. (4) gives the “functional performance equation” for ball milling

$$PRNP = TMP \times CSEff \times \text{LabGr} \times \text{BMGEff} \quad (6)$$

Equation (6) demonstrates that the production rate of the circuit is dependent on four factors. One is the power draw of the mill. Another is the nature of the ore in terms

of its grindability. It also shows that there is not just one “efficiency” but two active and distinct efficiencies involved in determining the circuit production rate. These are the “classification system efficiency” of the circuit, or the percentage of the mill energy used on coarse particles, and the “ball mill grinding efficiency,” which characterizes how well the energy being applied to the coarse particles is being utilized.

The functional performance equation was developed and named as an outcome of “value analysis and engineering” (Miles, 1972) of closed circuit grinding. Given the information outlined in the introduction to this paper, this process identified that the purpose of the grinding circuit is to generate as much new product with as little energy as possible. It then identified that the purpose of the equipment (pumps, cyclones and mill) was two-fold: first, classification to maximize the use of energy on coarse material, and secondly, the efficient size reduction of the coarse material by effective use of this energy.

### Industrial use and validation

The following are some examples from mineral-processing plants for which permission was granted to publish the data. They are intended to show how the functional performance equation is a tool that can be used to better understand and effectively improve grinding-circuit performance, as well as to demonstrate how the validity of this equation has been tested and assured.

**Sample calculation and dimensional analysis.** Table 1 presents data from the Les Mines Selbaie grinding circuit Survey No. 2, which was performed with the crusher fines stream off on a standard closed ball-milling circuit (Fig. 1). From the data, the work index performance of the circuit is calculated as follows:

- The work input (W) equals  $523 \text{ kW}/70.3 \text{ t/h} = 7.44 \text{ kWh/t}$ .
- $W = W_{Io} [(10/P80^2) - (10/F80^2)]$ .
- Solving:  $W_{Io} = 11.7 \text{ kWh/t}$ .
- The work index efficiency can be defined as the lab test work index divided by the operating work index:  $WIEff = 11.8/11.7 = 101 \text{ percent}$ .

The functional performance of the circuit is calculated as follows:

- The normal target P80 at this operation was  $106 \mu\text{m}$  (150 mesh) and was used as the basis for the calculations.
- The production rate of new product (PRNP) was calculated from the circuit tonnage and the percent minus  $106 \mu\text{m}$  (150 mesh) in the circuit feed and product:  $PRNP = 70.3 \text{ t/h} (77.6 \text{ percent} - 30.3 \text{ percent}) = 33.3 \text{ t/h}$ .
- The total mill power draw (TMP), measured at the pinion, was  $523 \text{ kW}$ .
- The ore grindability, from the Bond test, in this case was  $2.31 \text{ g/rev}$  (grams of new product per revolution of the Bond test mill).
- The CSEff is the average of the amount of plus  $106 \mu\text{m}$  material in the mill feed and discharge:  $CSEff = [(100 \text{ percent} - 21.8 \text{ percent}) + (100 \text{ percent} - 36.1 \text{ percent})] / 2 = 71 \text{ percent}$ .

Table 1

Data from the Les Mines Selbaie grinding circuit Survey No. 2, 1985.

Circuit feed rate:	70.3 t/h
Feed size: F80	1,160 $\mu\text{m}$
%-106 $\mu\text{m}$	30.3%
Bond test:	
W.I.	11.8 kWh/t
Grindability	2.31 g/rev
Cyclone overflow: P80	115 $\mu\text{m}$
%-106 $\mu\text{m}$	77.6%
Ball mill feed %: -106 $\mu\text{m}$	21.8%
Discharge	36.1%
Mill power draw:	523 kW

So far the functional performance equation for this survey is as follows:

- $PRNP = TMP \times CSEff \times LabGr \times BMGEff$ .
- $33.3 \text{ t/h} = 523 \text{ kW} \times 71.0 \text{ percent} \times 2.31 \text{ g/rev} \times BMGEff$ .
- Solving:  $BMGEff = 0.0388 \text{ (t/kWh)/(g/rev)}$ .

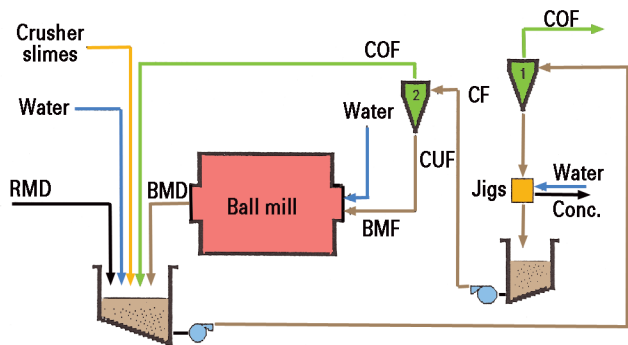
The units are the ratio of the specific grinding rate of coarse material in the ball mill in tons/kWh being applied to the coarse material over the grinding rate of coarse material in the lab mill in grams per revolution. This has also been termed the “grinding rate ratio.”

Finally, the functional performance equation for this plant experiment can be written as follows. For the Selbaie Baseline Survey No. 2 (1985) at  $106 \mu\text{m}$  (150 mesh):  $33.3 \text{ t/h} = 523 \text{ kW} \times 71.0 \text{ percent} \times 2.31 \text{ g/rev} \times 0.0388 \text{ (t/kWh)/(g/rev)}$ .

Verbally, during the survey the circuit was producing  $33.3 \text{ t/h}$  of new  $-106 \mu\text{m}$  ( $-150 \text{ mesh}$ ) product. It was doing so by applying 71 percent of the mill power to the coarse material, which had a lab grindability of  $2.31 \text{ g/rev}$ . The ball mill grinding efficiency (or grinding rate ratio between the plant mill and the test mill) was  $0.0388 \text{ (t/kWh)/(g/rev)}$ .

**A strategy for plant improvements.** Having written the outcome of a single, baseline plant survey, the strategy for improving grinding circuit performance becomes as follows: “to increase the values of CSEff and BMGEff by manipulating the variables that affect them.” Mill power draw variables (load level, speed, etc.) may also be examined if it is deemed desirable to do so.

Comparing survey data to those from other plants will offer information on where the best opportunities for improvement lie, whether in classification, grinding efficiency or both. Improving classification-system performance will lie with the pumps and cyclones and in reducing the cyclone overflow percent solids, if that is acceptable downstream. Key variables for improving mill grinding efficiency will be those associated with internal mill operating conditions, of which the grinding media sizing and percent solids come to mind. Engineering guidelines for media sizing (McIvor, 1997) and mill rheological conditions (Klimpel, 1984) can be examined to identify the best opportunities in this regard. Suitable tradeoffs can be reached when a given variable affects

**FIGURE 2****Simplified schematic of the Dome ball mill and circuit.**

both CSEff and BMGEff, for example, if the reduction in the amount of fines in the mill reaches the point where it negatively impacts grinding because of the resulting slurry rheology.

**Evaluation of different of grinding media.** Some years after the above test, Les Mines Selbaie undertook an investigation of different types of ball mill grinding media (McIvor et al., 1991). This led to a subsequent circuit survey for the evaluation of ball mill grinding efficiencies (mill percent solids was maintained constant), which is summarized below. For Selbaie New Media Grinding Survey, 1989, at 106  $\mu\text{m}$  (150 mesh): 32.1 t/h = 539 kW x 71.5 percent x 1.69 g/rev x 0.0493 (t/kWh)/(g/rev).

Comparison with the above results from baseline Survey No. 2 shows that the mill grinding efficiency increased from 0.0388 to 0.0493, or approximately 25 percent. During the 1989 survey, the work index efficiency was calculated to be 117 percent, or about a 16 percent relative increase. Each method confirms that a very significant improvement was achieved, although basic differences in the two models (Bond's 1952 third theory) dictate that they will not coincide quantitatively.

**Evaluation of a different classification system.** The Dome Mill grinding circuit employed coarse gold removal in the grinding circuit primary cyclone underflow. As a result of water addition in the coarse gold removal process, a second stage of cycloning was used on the primary cyclone underflow to raise the percent solids of the material feeding the ball mill (the secondary cyclone underflow) to a suitable level (Fig. 2). The primary cyclone overflow was very dilute, going to a thickener before carbon in pulp gold recovery. A survey was conducted on this circuit, including a Bond grindability test with a closing screen of 75  $\mu\text{m}$  (200 mesh). For comparison with the Selbaie circuit, the elements of the functional performance equation relative to each of the circuits' P80 product sizes were calculated as follows (McIvor et al, 1992):

- Functional performance of Dome Survey No. 1 (calculated at the actual P80 of 60  $\mu\text{m}$  or 250 mesh): 53.9 t/h = 865 kW x 85.5 percent x 1.32 g/rev x 0.0552 (t/kWh)/(g/rev).
- Functional performance of Selbaie Survey No. 2 (calculated at the actual P80 of 115  $\mu\text{m}$  or about 125 mesh): 33.7 t/h = 523 kW x 68.5 percent x 1.79 g/rev x 0.0525 (t/kWh)/(g/rev).

The classification system efficiency at Dome, with two-stage cycloning and high cyclone-feed water addition, is higher than that of Selbaie by a factor of approximately 25 percent. The grinding efficiency at Dome also calculated out to be slightly higher. With an operating work index of 8.5 kWh/t, compared to an ore work index of 11.5 kWh/t, the work index efficiency for the Dome survey calculated out to 136 percent, compared to 101 percent for Selbaie No. 2. Note that the constraint on total water addition before flotation at Selbaie made adoption of the same practice as Dome impractical.

**Evaluation of pebble mill operating percent solids.**

Following extensive preparations to maximize the quality of plant test data, surveys were carried out on the Tilden pebble milling circuit (identical in layout to the ball milling circuit in Fig. 1) over a period of several years. The first 14 plant tests were directed at the evaluation of a number of variables, such as grate discharge design, media (pebble) sizing and pump and cyclone adjustments. The last two were run at extremes of low and high mill feed water addition rates to explore the relationship between the pebble mill grinding efficiency and mill percent solids. The clarity of this trend was greatly enhanced by factoring out the relationship that was discovered to exist between mill grinding efficiency and the grindability of the mill feed itself (McIvor et al., 2000). The results are shown in Fig. 3. This led to the practice of operating the mills at increased percent solids, and yielded major energy cost savings.

Note that each point in Fig. 3 represents the grinding efficiency of coarse material in the mill, the last calculated element of the functional performance equation from each plant test. Although the ore, the mill power draw and the classification conditions varied, they were all accounted for to reveal the shown relationship. It is believed by the author that such results are unprecedented in grinding research.

**Summary and conclusions**

Since their inception, functional performance methods have been applied in 15 to 20 mineral processing plants. The results of some of these studies were published by the plant operators (Blythe, 1992). The author was closely involved in 10 such projects, several of which are described in the references. Others were carried out by individuals who became familiar with the method and then moved on to other operations.

Of those that the author is familiar with, several studies unexpectedly concluded that there was no effect from changes in certain design or operating variables. One audit showed that the circuit was operating near its potential peak of classification system efficiency. Two generated significant efficiency gains through mill-water optimization. Three studies produced benefits through use of more efficient grinding media. One of these was in open circuit ball milling, to which it can also be applied (McIvor et al., 1994). Four studies produced improvements in classification system efficiency through pump and/or cyclone modifications. Operating cost savings from these studies are measured in the millions of dollars annually. More studies are ongoing.

A grinding circuit is a complex system in that there are numerous interacting variables at work that affect

the output. The functional performance equation allows the observer to understand the system more clearly and to isolate the effect of different design and operating variables under his or her control. As these examples attest, it can make development and execution of a plan to improve and manage plant-grinding performance an effective exercise. Combined with other available tools — work index analysis, computer modeling and a systematic approach that also incorporates suitable metrics and process control — functional performance analysis provides the process engineers with a tool to take plant grinding operations to a new level of performance. ■

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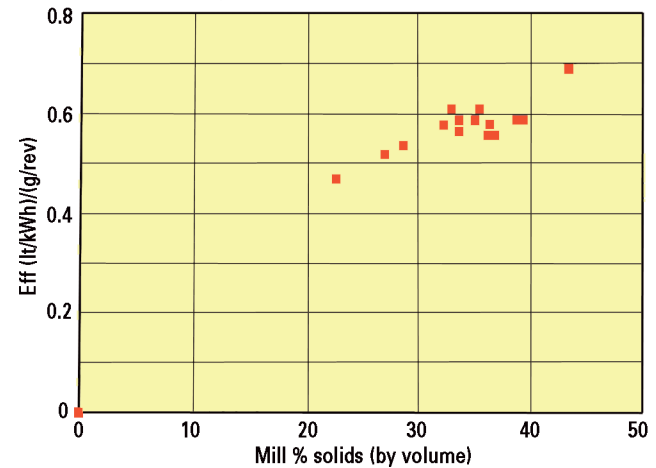
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**FIGURE 3**

**Mill grinding efficiency versus percent solids.**



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