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MODIFIED BOND AND RITTINGER ENERGY-SIZE RELATIONSHIPS FOR LABORATORY FINE GRINDING

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ABSTRACT – Laboratory-scale fine grinding tests remain the most practical method of assessing material grindability and predicting energy requirements for industrial scale fine grinding mills. This review describes several different fine grinding laboratory tests and analyses 88 test results. It was found that the original Bond and Rittinger energy-size models do not reproduce the experimental results with sufficient accuracy for fine grinding applications. Both the total average model error and standard deviation for individual laboratory data set results were high; mostly in the range of 15 - 20%. However, modified Bond and Rittinger energy-size models using a variable grindability coefficient were much more accurate. The total average model error and standard deviation reduced to mostly below 5%. This increased accuracy indicates potential to improve predictions of industrial scale energy requirements from fine grinding laboratory-scale tests using modified Bond and Rittinger energy-size models.

Keywords: Laboratory, Fine Grinding, Comminution, Energy, Size, Modelling.

INTRODUCTION

Fine grinding is becoming more common as unexploited deposits have increased complexity with reduced grain size and thus finer liberation size. The definition of fine and ultrafine grinding varies. However, grinding of material with an F80 (80% passing feed size) of less than 100 µm down to a P80 (80% passing product size) of 7 - 30 µm is generally considered fine grinding. While grinding of material with F80 of less than 20 µm down to P80 values with lower size limits of several µm is considered ultrafine grinding [1,2].

Ball mills have traditionally been used for fine grinding of minerals; however, efficiency decreases when grinding below 75 µm and ball mills are rarely economical below around 30 µm [4]. Fine grinding consumes higher specific energy than coarser grinds and is more efficiently achieved in alternative comminution devices with different dominant breakage mechanisms (attrition breakage rather than impact and abrasion). Stirred mills are typically used for fine grinding, including: tower mills, stirred media detritors and IsaMills™.

A tower mill is a low-speed vertical stirred mill where media motion is driven by the rotation of a helical impeller. It was invented in the 1950s by the Japan Tower Mill Company, and in the 1980s Metso introduced the Vertimill™, which is an adaptation of this technology. Almost exclusively steel grinding media is used with diameters typically at least 12 mm, up to several cm [5]. Sizing for industrial scale tower mills can be achieved using a pilot unit or Metso mostly uses a “jar test”, which

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is a batch ball mill test, for sizing the Vertimill™ [7].

The Stirred Media Detritor (SMD) is a medium-speed vertical stirred mill, which uses multi-layer pin impeller to fluidise the media (with media diameters typically of several millimeters) [5]. In the 1960s English China Clay (now Imerys) developed the SMD which originally used sand as media, and in recent decades sand was replaced with small ceramic grinding media beads. SMDs have been used in kaolin and calcium carbonate grinding applications, producing particle sizes down to 1–2 µm and Metso manufacture them for use in ultrafine grinding of metalliferous ores.

The IsaMill™ is a high-speed horizontal disc mill, developed by Mount Isa Mines Ltd. and Netzsch and commercialised for ultrafine grinding of ore in 1999 [8]. It consists of a horizontal cylindrical chamber, with the agitator being a series of perforated discs attached to a central shaft. It can produce very fine product sizes, with P80 values as low as 7 µm being practical [1], enabled by the relatively fine ceramic grinding media, in the range 1–8 mm in diameter [6].

Laboratory-scale fine grinding tests remain the most practical method of assessing material grindability and predicting industrial energy requirements. The Bond test is used for standard ball milling applications and the Levin test is a batch grinding test in the Bond mill developed for samples that are not conforming with the standard Bond procedure requirements. Specific tests and laboratory equipment are also available for the different stirred mill technologies. Examples of the fine grinding laboratory equipment used for the tests considered in this study are shown in Figure 1. The Levin test [10] is conducted using the standard Bond ball mill test equipment.

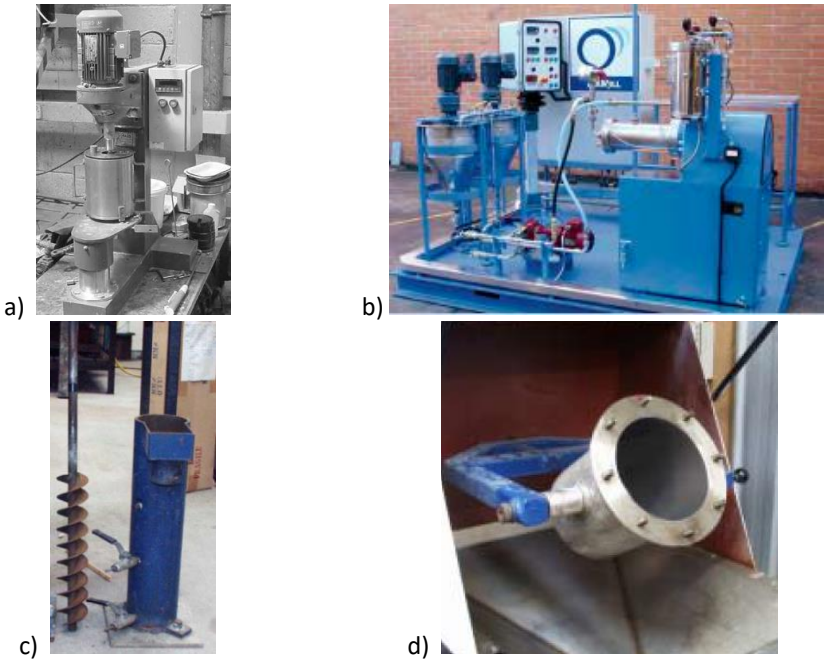

Figure 1 Laboratory fine grinding units a) Stirred Media Detritor (SMD), b) IsaMill™, c) tower mill, d) batch ball mill

EXPERIMENTAL

Test results from five different types of fine grinding laboratory tests from the authors data base were analyzed. The type of test, number of tests and data points from the test are summarized in Table 1. A total of 88 test results were analyzed with over 420 data points. IsaMill™, SMD and Levin tests are standardized tests and were conducted by commercial laboratories. The tower mill tests results are from the author's research work [9] and the batch ball mill test results were part of the authors consulting projects.

Table 1 Database of laboratory tests

Test type	Number of tests	Data points (energy-size pairs)
IsaMill Test	14	89
SMD Test	10	52
Tower Mill Test	40	187
Batch Ball Mill Test	14	48
Levin Test	10	47

For each grinding test the specific comminution energy (SE) was determined by measuring the power draw, or torque, or in the case of Levin test using the calibrated equations [10]. Most of the particle sizing was carried out using the laser sizing technology and only the Levin test used sieve sizing. Results for all tests are presented as product 80% passing size (P80) versus the specific energy (kWh/t) required to achieve this product size.

RESULTS AND DISCUSSION

To determine grinding energy requirements, well known Rittinger (1) and Bond (2) equations are commonly used to relate feed size, product size, and energy.

$$E_{1-2} = 10 * R\left(\frac{1}{x_2} - \frac{1}{x_1}\right) \quad \text{Von Rittinger} \quad (1)$$

$$E_{1-2} = 10 * B\left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}}\right) \quad \text{Bond} \quad (2)$$

Where:

$E_{1\text{-}2}$ = Specific grinding net energy to grind from particle size x_1 to x_2, kWh/t;

B, R = Constant of the respective equation (i.e., Bond and Rittinger constants are considered to be related to material properties and grinding conditions);

x_1, x_2 are the feed and product particle size respectively, that describes size distribution (80% passing size), microns.

Rittinger and Bond equations were applied to the energy-size data from all the tests listed in Table 1 and the constants were back-calculated for each energy-size pair. Factor 10 is used to conform with standard form of Bond equation.

Theoretically, for a set of grinding tests (same test type and ore type), the constant determined for each of the energy-size pairs should be the same; i.e., one constant should cover the whole test range. This was not the case for the test data analyzed in this study. The model constant for each test set was determined using a solver function to minimize the model error between the model energy output and the energy measured in the testwork. The constant determined for each test set was generally close to the mean of the back-calculated individual energy-size constants for that set of tests.

It was found that, for all the types of laboratory tests, the Bond and Rittinger energy-size models with fixed value constants R and B poorly predicted the experimental results. This was concluded based on the average error (Equation 3) and standard deviation (Equation 4) for each laboratory test set, as reported in the first two columns of Table 2.

$$\text{Average error} \qquad \mu = {\sum yi}/{n} \tag{3}$$

$$\text{Standard deviation} \qquad \delta = \sqrt{{\sum (yi - \mu)^2}/{n}} \tag{4}$$

Where yi are individual errors and n is the total number of data points for the laboratory test type.

It can be observed from Table 2 (first two columns), that the total average model error and standard deviation for individual laboratory data set results were high, mostly 15 - 20% for both Bond and Rittinger models with fixed value constants. Models with such high errors are not useful and the slightly lower errors from the Rittinger model are not worth further discussion.

Table 2 Average error and standard deviation of different forms of the Bond and Rittinger models

Average model errors/Standard Deviation of Model Errors (%)						
Model version	Fixed value Constant		Variable coefficient Linear		Variable coefficient Polynomial/Power	
Lab Test	Rittinger	Bond	Rittinger	Bond	Rittinger	Bond
Isamill	15.3/20.3	16.9/20.1	7.5/9.0	7.5/7.4	3.7/3.7	4.2/3.4
SMD	15.9/16.8	20.2/18.3	7.1/6.9	10.6/6.9	4.1/3.3	5.6/4.2
Batch Ball mill	17.2/18.2	22.0/20.4	10.6/14.8	13.5/13.5	8.5/3.8	8.6/4.6
Tower mill	9.6/11.2	11.0/12.1	4.8/5.9	4.4/4.2	2.7/2.9	2.6/2.9
Levin	14.2/17.4	13.4/15.2	6.0/5.6	6.4/5.5	3.1/4.2	3.4/4.0

Hukki [3] evaluated different energy-size grinding models (including Bond and Rittinger) and suggested that all models were valid, but each within a certain size range, as shown in Figure 2. Hukki observed that the energy requirements for breakage increase as particle size decreases and concluded that the exponent of the size terms in the energy-size relationship (or slope of the relationship) varies with size and material. Based

on this, it is not unexpected that the Rittinger model had lower errors for these fine grinding tests; however, the error is still too large for the model to be useful.

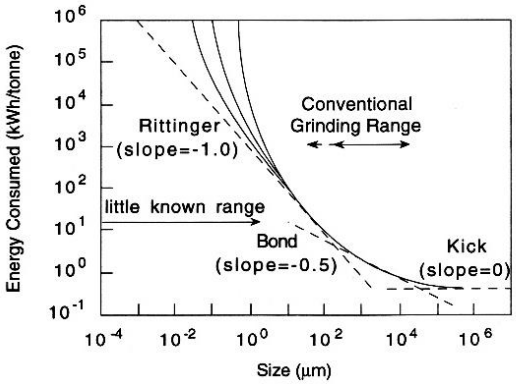

Figure 2 Relationship between energy input and particle size in comminution [3]

Considering the relationship between comminution energy and particle size, and to investigate reasons for poor predictions and how model error could be improved, the back-calculated set of constants were plotted against the product size (P80). Various "best fit" trends were found for each of the data sets, including linearly increasing and decreasing, polynomial and power trends. A small fraction of data had no trend. Some examples where the back calculated "constants" were strongly related to P80 are shown in Figure 3.

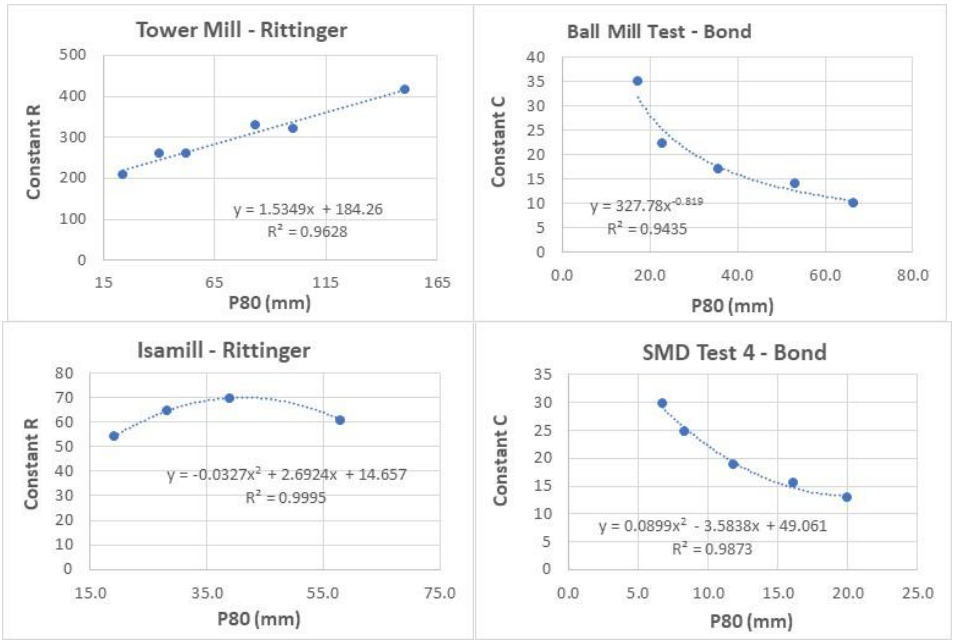

Figure 3 Examples of Bond and Rittinger constants dependance on product size P80

Based on the above analysis, it could be concluded that, for the laboratory tests data sets used in this study, the so called "constants" in Rittinger and Bond equations (Equations 1 and 2) are a function of grinding product size P80, see Equation 5.

$$B, R = f(P80) \tag{5}$$

Therefore, B and R are variable coefficients (rather than constants) and Rittinger (1) and Bond (2) equations could be modified accordingly to equations 6 and 7, respectively.

$$E_{1-2} = 10 * f_R(x_2)\left(\frac{1}{x_2} - \frac{1}{x_1}\right) \quad \text{modified Von Rittinger} \tag{6}$$

$$E_{1-2} = 10 * f_B(x_2)\left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}}\right) \quad \text{modified Bond} \tag{7}$$

Where:

x_1, x_2 are the product and feed particle size respectively, usually known as F80 and P80, µm

$f_R(x_2)$ Rittinger model variable coefficient, function of product size x_2.

$f_B(x_2)$ Bond model variable coefficient, function of product size x_2.

Applying the modified Bond and Rittinger equations improves the predictions significantly, as shown in Table 2. The results using a linear trend for the variable coefficients are shown in the third and fourth columns and with Polynomial/Power trends in the last two columns.

It can be observed that the total average model errors and standard deviations more than halved for both Bond and Rittinger models using linear variable coefficient compared to the models with fixed value constant. When the "best fit" variable coefficient trends (Polynomial/Power) were used, the errors were further reduced to below 5% in most cases (the average error was a bit higher for the Batch Ball mill test). Models are usually considered good if the errors are expected to be below 5%.

It is not possible to determine from this analysis which of the two models (modified Bond or modified Rittinger) may be better to use for fine grinding. Further analysis of the individual tests would be required to verify this. The results are expected to be published in follow up papers.

The experimental and Bond model predicted specific energy consumptions (kWh/t) for the SMD data set with the standard and modified models are compared in Figure 4. The error reduction can be clearly observed. Similar comparison was obtained for the other laboratory tests for both Bond and Rittinger models. The small errors confirm that the modified models could be used to describe the laboratory test results discussed in this work.

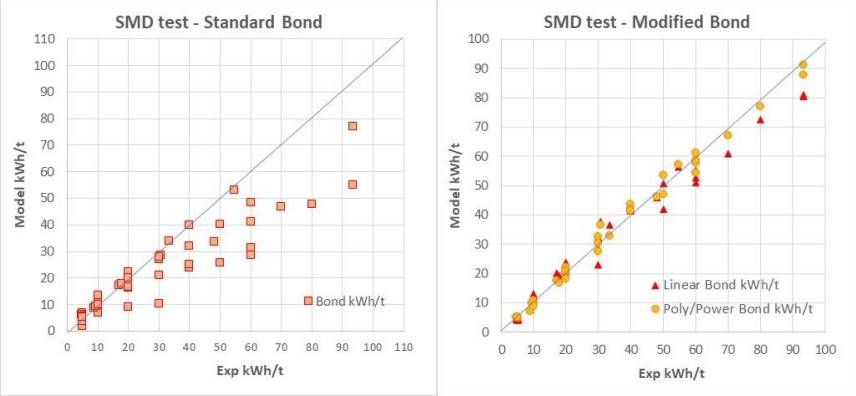

Figure 4 Standard and Modified Bond model predictions for SMD tests

CONCLUSION

Fine grinding consumes greater energy and often uses different technology (with different dominant breakage mechanisms) to coarser grinding applications. Bond and Rittinger size-energy relationships are commonly used for conventional coarser grinding applications, but when applied to data sets from fine grinding laboratory tests including Isamill, SMD, Levin, Tower mill and batch ball mill tests, the errors were high. The average model standard errors and standard deviations for each set of test data were in the range of 15 - 20%, and as such these models would not be usable.

It was found that the constants in the Bond and Rittinger models are strongly related to product size in most of the fine grinding cases investigated, and this was the major reason for poor model performance. Replacing the model constants with variable coefficients which are a function of product size dramatically improved the accuracy of the models, with errors reduced to mostly below 5% for both models. These small errors confirm that the modified Bond and Rittinger models with variable coefficient could be used to describe the fine grinding laboratory test results discussed in this work. Further analysis of the individual tests would be required to determine if either of the two models (modified Bond or modified Rittinger) is more suitable for modelling of fine grinding. The results of this analysis will be published in subsequent papers.

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